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# UNCERTAIN ENTRY MODELS, ENTRY BEHAVIOR, AND LIMIT PRICING*

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ABSTRACT

This paper applies the criterion of rational entry seen in recent certainty models of entry such as Selten (1975) and Friedman (1979) to the uncertain entry literature typified by Kamien and Schwartz (1971, 1975). In particular, we provide a decision theoretic model of entry for the ad hoc assumption, found in the uncertain entry literature, that entry occurs according to a Poisson process. A key implication of our analysis, not found in the uncertainty literature, is that post-entry profit of incumbents depends upon the pre-entry price. This implies that for an interesting nontrivial class of conjectural variations there is no limit pricing.

## 1. Introduction

A recent criticism of the classical limit price theory developed by Sylos (1969), Bain (1953) and Modigliani (1958) is its treatment, or rather its neglect, of the potential entrant's problem. (See, for instance, Needham (1969), Friedman (1979), and Selten (1975).) These authors all argue that in the context of the classical limit pricing model treating entrants as rational decision makers destroys the feasibility, let alone the rationality, of incumbents pursuing a pre-entry pricing (or output) policy to deter entry.

In view of this it is surprising that the entrant's behavior has continued to be treated in an ad hoc fashion in the uncertain entry literature. (See, for instance, Kamien and Schwartz (1971), Kamien and Schwartz (1975), Deshmukh and Chitke (1976), Lippman (1980), and finally Reynolds (1982) which shows Kamien and Schwartz (1975) and Deshmukh and Chitke (1971) to be special cases of a more general model.) In this paper we provide a decision theoretic foundation for the entry behavior assumed in the uncertain entry papers. The key assumptions we make are that there are costs associated with searching for entry opportunities, and that potential entrants differ according to their cost of production. These assumptions are sufficient, in equilibrium, to generate the arrival of entrants according to a Poisson process whose rate at t depends on the prevailing market price at t; the ad hoc assumption that entry occurs according to a Poisson process is common to all the uncertain entry papers cited above.

We show that while equilibrium limit pricing by incumbent firms is possible (for example, when firms conjecture that the output of other firms rises in proportion to their own) we argue that this only arises because we have allowed the conjectural variations of incumbents to be arbitrary. In particular, they admit the following pathology: firms with costs above the current market price may enter the market and survive only because they cause firms in the market to revise their conjectures about how market output changes with changes in their own output. To rule out such pathologies we argue that conjectures should satisfy the steadiness property. By this we mean that when a firm enters with unit cost of production above the current market price, it should simply be ignored by incumbents. When conjectures satisfy steadiness, limit pricing is an irrational action for incumbents to pursue.

This result is in contrast to the usual result of the uncertain entry literature, that, without regard to what conjectural variations are assumed, limit pricing is an optimal policy for incumbents to follow. As can be observed from our theorem, this conflict follows from the fact that in the uncertain entry literature, all entrants are assumed to be identical. Our results hinge on the steadiness property and the fact that with possible entrants, distributed according to their costs, there is a marginal type of entrant to deter.

The paper is organized as follows. In Section 2 we provide our model of entry behavior. In Section 3 we use the Bertrand conjectures model as an example where limit pricing does not occur. In Section 4 a general conjectural variations model is introduced. It is shown that when conjectural variations satisfy the steadiness property, limit pricing is irrational. Section 5 concludes the paper.

## 2. Entry

A universal assumption in the uncertain entry literature cited in Section 1 is the assumption that entry into a market for a quantity x of some good occurs according to a non-homogeneous Poisson process (see Ross (1970)) whose rate h at time t depends on the prevailing market price at t. It is assumed h(p) is a differentiable, non-negative increasing function of p. The probability of entry by date t is

$$1 - \exp[\int_0^t h(p(s))ds] \quad . \tag{1}$$

The purpose of this section is to provide a decision-theoretic foundation for entry behavior sufficient to explain the entry assumption of the uncertain entry models. This objective is consistent with the current trend in the limit pricing literature as a whole. (See, for example, Needham (1969), Friedman (1979), Selten (1975), and Milgrom and Roberts (1982A, 1982B).)

Suppose there is a large number of independent markets, and a large supply of entrepreneurs who may assign themselves to (enter) any number of these markets. In the absence of incomplete information or increasing costs of shifting resources, long-run equilibrium and short-run equilibrium coincide: in other words the world we imagine is a purely static one. Consequently, to justify the entry behavior assumed in the uncertain entry literature a friction of some kind must be introduced.

We assume that, while each entrepreneur may know the distribution of investment opportunities, he must sample to determine the climate for investment in any particular market, and determine his talent for producing in this market. We assume there are increasing costs in sampling rates. That is, we have in mind a search problem. However, unlike a standard search model, there is no opportunity cost in entering a market in terms of foregoing the opportunity to search further and to enter more markets. When such an entrepreneur arrives at market x, we call him a potential entrant. Due to the increasing costs of sampling, they choose a finite sampling rate. Assuming market x to be an infinitesimal portion of overall investment opportunities, the aggregate rate of sampling is invariant to conditions prevailing in market x. In particular, this implies that the arrival of potential entrants occurs according to a homogeneous Poisson process with rate $\lambda_o$. (Formal details are in Appendix A.)

Potential entrants will enter if they expect positive profits. Suppose, for the moment, that the entry decision depends solely on the price prevailing at the time of arrival and the potential entrant's cost of production. If all firms are identical, as is assumed in the uncertain entry literature, then

$$h(p) := \begin{cases} \lambda_o & p > p^* \\ 0 & p \leq p^* \end{cases}$$

where $p^*$ is the minimum price at which entry is considered profitable.

In order to justify the assumption that $h(p)$ is a differentiable, non-negative, increasing function we assume that the firms arriving in market x are not identical. In particular, each potential entrant has a constant unit cost of production. Production costs are continuously distributed according to $F(c)$. We let $f(c) := F'(c)$. Suppose, for the moment, that a potential entrant with cost c enters when $q(p) > c$, where $q(p)$ is a non-negative, increasing function of p. Then

$$h(p) = \lambda_o F(q(p))$$

so that the density of entry (see Ross (1970)) at time t is:

$$e^{-\Phi(t)}\ \Phi'(t) \tag{2}$$

where $\Phi(t) := \lambda_o \int_o^t F(q(p(s)))ds$ .

As Friedman (1979), for one, has argued, a fully informed rational potential entrant will make his entry solely on the basis of his profits after entry, and that an entrant's profits are entirely independent of pre-entry price, in particular. Friedman has in mind the two period world in which there is a single potential entrant in the second period. Obviously, here limit pricing is not even feasible, let alone optimal. This argument extends to a finitely repeated game as well: see Selten (1975).

To permit the feasibility of limit pricing by incumbent firms we assume potential entrants use the function $q(p)$ in their entry decision. There are at least two ways of rationalizing this.

First, the potential entrant may not be able to directly observe the costs of incumbents in the market and can only infer them from the prevailing market price. Milgrom and Roberts (1982A) use a similar assumption in a two period model where a potential entrant is known to appear in the second period. As in Milgrom and Roberts (1982A), $q(\cdot)$ is determined in equilibrium so that no potential entrant is fooled.

A second rationalization for $q(p)$ follows from the infinitely repeated nature of the game. As noted, Selten's argument applies to finitely repeated games: as can be shown (see, for instance, Milgrom and Roberts (1982B), Appendix A), entry deterring behavior is possible in an infinitely repeated game. In our model, each time a firm enters, each firm resolves its initial problem taking account of the new firm. Except for the addition of a new firm, each incumbent faces the same problem as before.

Due to the fact there is no last entry (i.e., the game is infinitely repeated), $q(\cdot)$ might rationally depend on p, among other things such as the costs of the incumbents, even when the potential entrant knows the costs. Since the threat of entry exists after a new entrant comes in, a "low" price may indicate that incumbent firms are attempting to deter entry, and hence a lower post entry price is expected. It is conceivable that entry deterrence is Nash equilibrium behavior, in the sense that if incumbents believe in $q(p)$, potential entrants should as well, and vice versa. In any case, such a $q(p)$ must be hypothesized if the possibility of limit pricing is to be tested.

## 3. An Example

Consider a market for a homogeneous product. Market demand is $x(p)$, where p is price. We assume $x(p) \cdot p$ is strictly concave and that $x(p) \geq 0$ for all $p \geq 0$. Initially, there are n firms in the market. Consistent with the assumptions of Section 2, the $i^{th}$ firm has a constant, positive unit cost of production of $c_i$, $i=1,\ldots,n$. It is assumed that each firm sets price, taking the prices of its rivals as given. In the absence of the threat of entry, that is, in a static game, this is a Bertrand pricing game with undifferentiated products. Without loss of generality, for the argument of the present section, the $c_i$ can be ordered so that $c_i \leq c_{i+1}$ for $i=1,\ldots,n-1$.

As is well known, given the cost functions assumed, there is a solution to this game. The only firms of any consequence are firm 1 and firm 2. Firm 1 is the only producer as long as his price does not exceed $c_2$, the smallest price firm 2 would ever offer. Firm 1 sets p, the market price, to maximize his profits, $\pi(p;\, c_1,c_2)$, defined by

$$\pi(p;\, c_1,c_2) := \left\{ \begin{array}{ll} x(p)[p-c_1], & p \leq c_2 \\ 0 & , p > c_2 \end{array} \right\} \quad .$$

Define

$$p^m(c_1,c_2) := \operatorname{argmax} \pi(p;\ c_1,c_2)$$

That is, $p = p^m(c_1,c_2)$ satisfies

$$\begin{cases} p = c_2 \qquad \text{if } \pi'(p;\ c_1,c_2)\Big|_{p=c_2} > 0 \\ \text{and} \\ \pi'(p;\ c_1,c_2) = 0 \quad \text{otherwise}\ , \end{cases} \tag{3}$$

where $\pi'(p;\ c_1,c_2) := x(p) + x'(p)(p-c_1)$. The equilibrium price in the static Bertrand game provides a benchmark for comparison to the price in the dynamic Bertrand game with uncertain entry. If price in the dynamic game is less than $p^m(c_1,c_2)$, then limit pricing is said to occur.

More generally, any response at all to the threat of entry can be called limit pricing. That is, if the threat of entry induces a higher price, we should call this limit pricing (or perhaps anti-limit pricing) as well. This will not occur in normal environments, but such pathologies can be constructed (see Appendix B).

In Section 2 an entrepreneur arriving at the market at date t with cost c entered if and only if $q(p) > c$. In this section we specify $q(p) = p$. This specification can be justified in at least three ways. First, potential entrants follow the same Bertrand behavior as firms in the market. (See Novshek (1980), Definition 5, for a similar assumption.) If $p(t) < c$ an entrepreneur of cost c arriving at date t takes this price as given and decides it is not worthwhile to make an offer in this market. Second, $q(p) = p$ turns out to be rational in the sense that no potential entrant decides not to enter when it could have made a profit. Finally, the entry decision may in fact be rational in Friedman's

(1979) sense, but the incumbent firms may believe $q(p) = p$. In this case, it turns out that given the incumbent's reaction to $q(p) = p$, entry occurs in just the way predicted. In other words, this is a Nash equilibrium in the sense that no one's beliefs are contradicted.

From equation (2), when $q(p) = p$, the probability of a firm entering at time t is

$$\lambda_o F(p(t)) \exp[-\lambda_o \int_o^t F(p(s))ds] .$$

The probability of a firm of cost $c \leq p(t)$ entering at time t is

$$\frac{f(c)}{F(p(t))} \lambda_o F(p(t)) \exp[-\lambda_o \int_o^t F(p(s))ds] .$$

Thus, the (expected) value to firm 1 of being the least cost firm is

$$V(c_1,c_2) : =$$

$$\max_{p(t)} \int_o^\infty [ \int_o^t e^{-rs}\pi(p(t);\ c_1,c_2)ds$$

$$+ \int_o^{p(t)} e^{-rt} V(c_1,c\,) \frac{f(c)}{F(p(t))} dc]\ \lambda_o F(p(t)) \exp[-\lambda_o \int_o^t F(p(s))ds]dt,$$

where r is the constant positive rate of discount. Note that $V(c_1,c_2)$ is defined recursively: when entry of a firm of cost c at time t occurs, firm 1's (expected) value becomes $V(c_1,c)$ defined in the same way as $V(c_1,c_2)$. That is, there is no end to the entry threat. Note $V(c_1,c) : = 0$ when $c \leq c_1$. It should be obvious that the optimal $p(s)$ is constant up until the actual date of entry. (See Lemma 1 in Section 4.) Thus, we can write

$$V(c_1,c_2) : = \max_p W(p;\ c_1,c_2),$$

where

$$W(p;\ c_1,c_2) : = \frac{\pi(p;\ c_1,c_2) + \int_o^p V(c_1,c)f(c)dc}{r + \lambda_o(p)} .$$

Define

$$p^F(c_1,c_2) : = \text{argmax } W(p;\ c_1,c_2) .$$

That is, $p = p^F(c_1,c_2)$ satisfies

$$\begin{cases} p = c_2 \quad , \qquad \text{if } \left.\dfrac{dW}{dp}(p;\, c_1,c_2)\right|_{p=c_2} > 0 \\ \text{and} \\ \dfrac{d}{dp}W(p;\, c_1,c_2) = 0 \quad , \quad \text{otherwise} \end{cases} \tag{4}$$

where

$$\frac{dW}{dp}(p;\, c_1,c_2) =$$

$$\frac{1}{r+\lambda_o F(p)}\left[\pi'(p;\, c_1,c_2) + V(c_1,p)f(p) - \frac{[\pi(p;\, c_1,c_2) + \int_o^p V(c_1,c)f(c)dc]f(p)}{r+\lambda_o F(p)}\right].$$

Using the definition of $W(p;\, c_1,c_2)$,

$$\frac{dW}{dp}(p;\, c_1,c_2) = \frac{1}{r+\lambda_o F(p)}\left[\pi'(p;\, c_1,c_2) + [V(c_1,p) - V(c_1,c_2)]f(p)\right]. \tag{5}$$

Note that if $p = c_2$, then $V(c_1,p) = V(c_1,c_2)$. Moreover, if $p < c_2$, then it is clear that, for $p \le c \le c_2$, $V(c_1,c) = V(c_1,c_2)$. Consequently, $V(c_1,p) = V(c_1,c_2)$ and equation (5) simplifies to

$$\frac{dW}{dp}(p;\, c_1,c_2) = \frac{\pi'(p;\, c_1,c_2)}{r+\lambda_o F(p)} \tag{6}$$

<u>DEFINITION</u>: Limit Pricing. Limit pricing is said to occur when

$p^m(c_1,c_2) > p^F(c_1,c_2)$.

<u>PROPOSITION</u>: <u>NO LIMIT PRICING</u>. $p^m(c_1,c_2) = p^F(c_1,c_2)$.

<u>PROOF</u>: $p^m(c_1,c_2)$ is defined by equation (3). $p^F(c_1,c_2)$ is defined by equation (4). In view of equation (6) they are equivalent. ■

This result has a very appealing noncalculus explanation. In order to deter a firm with costs $c < c_2$, firm 1 must set $p \le c$. But this implies that his profit before (and after) one such potential entrant arrives is no more than his profit had he set $p > c$ and the firm entered at the date it arrived. Such a firm can be deterred, lowering the probability of entry at each date, but only at the cost of foregone pre-entry profit, without any offsetting gain.

This example is similar in many respects to Kamien and Schwartz (1971), henceforth KS, which may be the simplest of the uncertain entry models cited in the introduction. The major difference between this example and KS is that we provide an explicit decision making model for the behavior of entrants, consistent with the ad hoc assumptions on entry made by KS. Our model of entry behavior developed in Section 2 forced us to recognize that not only will the probability of entry depend upon price, but also the types (in terms of unit costs) of firms that enter will be affected by price.

Relative to the KS model, our innovation was to recognize that the post-entry value of firm 1,

$$\int_0^p V(c_1,c)\,\frac{f(c)}{F(p)}\,dc \ ,$$

is not independent of the pre-entry price. KS assumes post-entry profits are exogenously given and equal to $\bar{\pi}$, so that the post-entry value of firm 1 is $\bar{V}:=\frac{\bar{\pi}}{r}$. It is assumed $\bar{\pi} < \pi(p^m(c_1,c_2);\ c_1,c_2)$. This is a simplification of our model where

$$V(c_1,c_2) := \max_p W(p,c_1,c_2)$$

$$:= \max_p \left\{ \frac{\pi(p,c_1,c_2) + \frac{\bar{\pi}}{r}F(p)}{r+\lambda_o F(p)} \right\}$$

(Compare with equation (27) of KS.) For $c_2$ sufficiently large, the optimum price occurs at

$$0 = \frac{dW}{dp}(p;\, c_1,c_2)$$

$$= \frac{1}{r+\lambda_o F(p)}\left[\pi'(p;\, c_1,c_2) + \frac{\bar{\pi}}{r} f(p) - \frac{[\pi(p_1,c_1,c_2) + \frac{\bar{\pi}}{r}F(p)]f(p)}{r+\lambda_o F(p)}\right]$$

$$= \frac{1}{r+\lambda_o F(p)}\left[\pi'(p;\, c_1,c_2) - \frac{(\pi(p;\, c_1,c_2) - \bar{\pi})}{r+\lambda_o F(p)} f(p)\right]$$

(Compare with equation (13) of KS.) Limit pricing follows directly: p equals the myopic price $p^m(c_1,c_2)$ if and only if $f(p^m(c_1,c_2)) = 0$.

The reader may wonder whether our non-limit pricing result depends upon our assumption that the entry process never terminates (in contrast to the KS assumption that entry occurs once (or a known number of times)). In fact, when entry is limited to one firm we find pricing to enhance rather than to limit the probability of entry. This is described in Appendix B.

4. The Model

In the preceding section, the notions of limit pricing and the function q were clearly defined. In this section, we shall extend the notion of limit pricing to apply to more general structures. Initialize the model with n firms, $n \geq 1$, with costs $c_1,\ldots,c_n$. Let $\bar{c} = (c_1,\ldots,c_n)$. Often we shall suppress $\bar{c}$ for clarity. Let the output of the $j^{th}$ firm be $x_j^n$ and $x^n = \sum_{j=1}^{n} x_j^n$.

A very general model of market behavior is obtained by defining conjectural variations as follows. Suppose firm j believes that[1]

$$\frac{\partial x^n}{\partial x_j^n} = \beta_j^n(\bar{c}) \tag{7}$$

The central question in the uncertain entry literature is whether incumbents expand their output in the presence of the threat of entry. That is, whether there is limit pricing. Thus, limit pricing occurs only if the equilibrium conditions of the dynamic problem, with entry possible, differ from the equilibrium conditions of the no entry (static) oligopoly problem. (The converse does not follow: there can be anti-limit pricing.) The equilibrium conditions of the no entry problem are quite easy to generate.

The $j^{th}$ firm faces the optimization problem

$$\max_{x_j^n}(p(x^n) - c_j)x_j^n \quad \text{subject to } x_j^n \geq 0 \text{ and (7).} \tag{8}$$

$p(x^n)$ is assumed to be a continuously differentiable inverse demand function. The first order conditions for (8) are:

$$p(x^n) - c_j + x_j^n p'(x^n)\beta_j^n = 0, \text{ if } p(\sum_{i \neq j} x_i) - c_i > 0 \tag{9}$$

and $x_j^n = 0$ otherwise. Define $p_n(c_1,\ldots,c_n)$ to be the equilibrium price solving (9).

In equilibrium, we assume that rational entry corresponds to the condition that any firm with cost less than price after entry enters. Consequently, in the notation above, equilibrium requires:

$$q(p_n) = p_{n+1}\ (\bar{c}, q(p_n)) \tag{10}$$

Let $v_j^n(\bar{c})$ be the discounted expected profit accruing to the $j^{th}$ firm in the n firm dynamic oligopoly. Clearly $v_j^n(\bar{c})$ is composed of profit earned prior to the entry of the $n+1^{st}$ firm plus the expected value of $v_j^{n+1}(\bar{c},c_{n+1})$, discounted by the time of entry. If r is the discount rate, we obtain:

$$v_j^n(\bar{c}) = \max_{x_j^n(s)}\left\{\int_o^\infty \left[\int_o^t e^{-rs}(p(x^n(s)) - c_j)x_j^n(s)ds + \int_o^{q(p(x^n(s)))} e^{-rt}v_j^{n+1}(\bar{c},c)\frac{f(c)dc}{F(q(p(x^n(t))))}\right]e^{-\Phi(t)}\,\Phi'(t)dt\right\} \tag{11}$$

where $\Phi(t) = \lambda_o \int_o^t F(q(p(x^n(s))))ds$, as in (2).

Lemma 1: $x_j^n(t)$ is constant.

Proof: This is a consequence of the Bellman Principle of Optimality and the observation that $x_j^n(t_o+t)$ solves a linear transformation of the problem $x_j^n(t)$ solves. Thus, for $t_o \geq 0$, $x_j^n(t) = x_j^n(t_o+t)$.

Lemma 2:

$$v_j^n(\bar{c}) = \max_{x_j^n} \left\{ \frac{(p(x^n)-c_j)x_j^n + \lambda_o \int_o^{q(p(x^n))} v_j^{n+1}(\bar{c},c)f(c)dc}{r + \lambda_o F(q(p(x^n)))} \right\}$$

Proof: Straightforward, given Lemma 1, and integration of (11).

Lemma 3: The first order conditions for dynamic optimization:

$$p(x^n) - c_j + x_j^n p'(x^n)\beta_j^n =$$

$$\lambda_o[v_j^n(\bar{c}) - v_j^{n+1}(\bar{c},q(p(x^n)))]f(q(p(x^n)))q'(p(x^n))p'(x^n)\beta_j^n. \qquad (12)$$

By Lemma 3 and (9), we have limit pricing when the right-hand side of (12) is nonzero. The case $\lambda_o = 0$ is the situation when no entrepreneurs visit the x market, and in this case (9) coincides with (12), trivially. A second trivial case occurs when $q'(p) = 0$; this case was considered in Section 2.

$\lambda_o = 0$ and $q'(p) = 0$ are the cases when limit pricing is ineffective; naturally it doesn't occur then. The interesting case is when limit pricing is effective at slowing the rate of entry, and is nevertheless suboptimal. We contend the following theorem demonstrates that limit pricing is a result of pathological conjectural variations at best. We presume $\lambda_o > 0$ and $q' > 0$ in the solution of (12).

THEOREM: If $\beta_j^n(\bar{c}) = \beta_j^{n+1}(\bar{c},p_n(\bar{c}))$, then a solution to (9) is a solution to (12).

Proof: The proof proceeds in two parts. First we show that, in the static game, the entry of a firm with cost $p_n(\bar{c})$ to the n firm market does not affect the price. We then show this implies $v_j^n(\bar{c}) = v_j^{n+1}(\bar{c},p_n(\bar{c}))$, sufficient in view of Lemma 3.

(i): Consider the n+1 firm equilibrium when $c_{n+1} = p_n(\bar{c})$. We obtain

$$p_{n+1} - c_j + x_j^{n+1} p'(x^{n+1})\beta_j^{n+1} = 0 \qquad j=1,\ldots,n \tag{13}$$

$$p_{n+1} - c_{n+1} + x_{n+1}^{n+1} p'(x^{n+1})\beta_{n+1}^{n+1} = 0 \tag{14}$$

By (14) and the hypothesis that $\beta_j^n(\bar{c}) = \beta_j^{n+1}(\bar{c},p_n(\bar{c}))$, the n equations of (13) are identical to (9). Consequently, (13) is satisfied by $p_n = p_{n+1}$. As $c_{n+1} = p_n$, (14) is satisfied by $p_n = p_{n+1}$ and $x_{n+1}^{n+1} = 0$. Thus, a solution to (9) also solves (13) and (14), with price constant.

(ii): Note that in this case, $q(p) = p$ is a solution to (13) and (14). Indeed if the solution is unique, $q(p) = p$ is a unique solution to the static game.

To complete the proof, we show $v_j^{n+1}(\bar{c},q(p)) = v_j^n(\bar{c})$.

When a firm with cost $p_n(\bar{c}) = q(p_n)$ is added to the first n firms, we observed the price remains constant. Thus the profit in the n firm case prior to the entry of firm n+1 is identical to the profit in the n+1 firm case $(c_{n+1}=p_n)$ prior to the entry of the firm n+2.

In addition, profits after entry are identical as well. Note that if $q(p) = p$, then

$$p_{n+1}(\bar{c}, c_{n+1}) \le p_{n+1}(\bar{c}, p_n) = p_n$$

and thus price is not increasing. Consequently, the entry of firm n+2 into the market with firms of costs $\bar{c}$ and $p_n$ drives out the n+1$^{st}$ firm with probability one. Therefore, the result of entry into the n+1 firm environment ($c_{n+1}=p_n$) corresponds to the result of entering an n+1$^{st}$ firm into the n firm environment. Thus, profit of the j$^{th}$ firm is invariant to the addition of a firm with cost $p_n$. This proves $v_j^n(\bar{c}) = v_j^{n+1}(\bar{c}, q(p))$. From Lemma 3, this forces a solution to (9) to solve (12). ■

<u>Corollary</u>: In the no limit pricing solution of (12), if $\beta_j^n(\bar{c}) = \beta_j^{n+1}(\bar{c}, p_n(\bar{c}))$, then $q(p) = p$.

We shall call an oligopoly with conjectural variations that satisfy $\beta_j^n(\bar{c}) = \beta_j^{n+1}(\bar{c}, p_n(\bar{c}))$ to be steady, and refer to this case as a steady game. Two important steady games are Nash ($\beta_j^n=1$) and perfect competition ($\beta_j^n=0$). Essentially, we have shown that steady oligopolies do not limit price. It is rather easy to see that if an oligopoly never limits prices for all F and $p(x)$, then it is steady as well.

We shall propose two views of steady oligopolies. From a conjectural variations viewpoint, the steadiness property is a measure zero accident, forcing a severe restriction on the possible $\beta_j^n$ values. If one feels that a firm's conjectural variation is an arbitrary choice (perhaps satisfying $0 \le \beta_j^n \le n$, perhaps not), then steadiness is improbable at best. Indeed, if a conjectural variation is a constraint on the firm, like a production function, then steadiness is unlikely, and hence some limit pricing likely.

The authors do not hold with this view, and contend that it is no accident that the two most analyzed behavioral hypotheses, Nash and perfect competition, are steady. In order to analyze the plausibility of a set of conjectural variations, one must appeal to a different analytic approach. Considering the steadiness property from a game theoretic view, it appears quite plausible. To see this view, consider n firms solving respective optimization problems, and arriving at a price p. They may or may not be behaving as a coalition, but they are optimizing, and p is, in some way, the optimum price at which they arrive. Now, if an $n+1^{st}$ firm is added with cost p, the price p is still feasible, and firm n+1 is an ineffective constraint. If the first n firms ignore the $n+1^{st}$ entry, he will not be able to block their behavior. The steadiness property is precisely the act of ignoring ineffective entries, and seems quite plausible from this view. Indeed, for an $n+1^{st}$ firm with cost $p_n$ to stay in, $p_{n+1}$ must exceed $p_n$, which means the first n firms cut their output to accommodate firm n+1. Certainly there are conjectural variations that support such behavior; indeed $\beta_j^n = n$ will. However, this behavior is difficult to rationalize from a game theoretic viewpoint. Note that steadiness can be considered the "irrelevance of irrelevant firms".

## 5. Conclusion

The uncertain entry literature takes it as a fact that entry occurs randomly in time. In particular, it assumes entry occurs according to a nonhomogeneous Poisson process with rate h(p). We sought to explain this from an analysis of the entry decision. We assumed potential entrants appear randomly in time at a fixed rate. In view of this assumption the random appearance of actual entrants requires that the potential entrants differ in some respect. To satisfy this requirement we assumed that the unit cost of a potential entrant is a draw from a distribution. This forced the post-entry environment to depend on this distribution of costs. In particular, when entrants are not identical, the value of a firm after the addition of another firm to the market is a random variable because the ex ante unit cost of the new firm is a random variable. Common throughout the uncertain entry literature is the assumption that, once in the market, all firms are identical.

To investigate the consequences of the assumption that potential entrants were not identical, we first considered a model of price setting Bertrand firms. This analysis resembles the seminal work in this literature, Kamien and Schwartz (1971). It was shown that pricing to deter entry, while feasible, was suboptimal from the incumbents' point of view. The explanation for this was quite simple: to deter a particular cost firm, the incumbents' profit must be the same as had that type of firm entered.

As Reynolds (1982) shows, Kamien and Schwartz (1975) and Deshmukh and Chitke (1976) are special cases of a more general conjectural variations model. In particular, Kamien and Schwartz (1975) make the Cournot assumption that each incumbent choose output taking the other incumbents' output as given; Deshmukh and Chitke (1976) assume each incumbent believes the output of each other producer will increase in proportion to its own. In Reynolds (1982) these models are special cases. Each of these papers concludes that incumbents will produce beyond the static equilibrium output (limit pricing). We showed that when potential entrants are not identical, limit pricing is not necessarily a property of these conjectural variations models. In particular, we argued that it is reasonable to assume that if a firm entered the market with unit costs above the current price, incumbents should act as if the new firm did not exist. This is equivalent to saying their conjectures of the rate of change of market output, with respect to their own, should remain unchanged. When this steadiness property is satisfied there is no limit pricing.

## Footnotes

*We wish to thank Ignatius Horstmann, Glenn MacDonald, and John McMillan for helpful comments. Any errors are our own.

[1] In many problems, it will be the case that the entry history determines equilibrium and affects the conjectural variations. Equation (7) allows for this general case, but does not rule out the usual much stronger assumption that

$$\beta_j^n = \beta(n), \ \forall j,$$

nor does it rule out symmetric forms of $\beta_j^n$ such as

$$\beta_j^n = \beta^n(c_j; \bar{c}) .$$

## References

BAIN, J. S. (1953), Barriers to New Competition (Cambridge).

DESHMUKH, S. D. and CHITKE, S. D. (1976), "Dynamic Pricing with Stochastic Entry," Review of Economic Studies 43, 91-97.

FRIEDMAN, J. (1979), "On Entry Preventing Behavior," in Applied Game Theory, ed. by S. J. Brams, A. Schotter, and G. Schwodiauer, (Wurzburg, Vienna: Physica-Verlag), 236-253.

KAMIEN, M. I., and SCHWARTZ, N. L. (1971), "Limit Pricing and Uncertain Entry," Econometrica 39, 441-454.

KAMIEN, M. I., and SCHWARTZ, N. L. (1975), "Cournot Oligopoly with Uncertain Entry," Review of Economic Studies 42, 125-131.

LIPPMAN, S. A. (1980), "Optimal Pricing to Retard Entry," Review of Economic Studies 47, 723-731.

MILGROM, P. and ROBERTS, J. (1982A), "Limit Pricing and Entry Under Incomplete Information: An Equilibrium Analysis," Econometrica 50, 443-459.

MILGROM, P. and ROBERTS, J. (1982B), "Predation, Reputation, and Entry Deterrence," Journal of Economic Theory 27, 280-312.

MODIGLIANI, F. (1958), "New Developments on the Oligopoly Front," Journal of Political Economy 66, 215-232.

NEEDHAM, D. (1969), Economic Analysis and Industrial Structure, Ch. 7 (Holt, Rinehart and Winston).

NOVSHEK, W. (1980), "Cournot Equilibrium with Free Entry," Review of Economic Studies 47, 473-486.

REYNOLDS, S. (1982), "Limit Pricing, Conjectural Variation, and Entry," Economics Letters 9, 195-199.

ROSS, S. M. (1970), Applied Probability Models with Optimization Applications (Holden -Day).

SELTEN, R. (1975), "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," International Journal of Game Theory 4, 25-55.

SYLOS-LABINI, P. (1969), Oligopoly and Technical Progress (Cambridge, Mass.).

# APPENDIX A

Suppose entrepreneurs have types $\gamma$, where $\nu$ is the p.d.f. of $\gamma$. The type $\gamma$ determines their average post-entry profit in a market, defined by $\mu(\gamma)$. The firms search at rate $\alpha$, with cost $\xi(\alpha,\gamma)$. $\xi$ is convex increasing in $\alpha$, with $\frac{\partial\xi}{\partial\alpha}(0,\gamma) = 0$ and $\frac{\partial^2\xi}{(\partial\alpha)^2}(\alpha,\gamma) \geq \xi_o(\gamma) > 0$ for all $\alpha$.

An entrepreneur of type $\gamma$ maximize expected profits:

$$E\pi(\gamma) = \int_o^\infty e^{-rt}\left[\frac{\mu(\gamma)}{r}\right]e^{-\int_o^t \alpha(s)ds}\alpha(t)dt - \int_o^\infty e^{-rt}\xi(\alpha(t),\gamma)dt$$

$\alpha$ is constant, from the Bellman Principle of Optimality. Thus:

$$E\pi(\gamma) = \left[\frac{\mu(\gamma)\alpha}{r+\alpha} - \xi(\alpha,\gamma)\right]\frac{1}{r}.$$

First order optimization conditions force:

$$\frac{\mu(\gamma)r}{(r+\alpha)^2} = \frac{\partial}{\partial\alpha}\xi(\alpha,\gamma)$$

By the hypotheses on $\xi$, a unique solution $\alpha(\gamma) > 0$ exists, and this maximizes $E\pi(\gamma)$, owing to the concavity of $E\pi$. Finally, the rate of appearance of entrepreneurs to a particular market is linear in the rate entrepreneurs search, forcing

$$\lambda_o = \lambda\int\alpha(\beta)\nu(\beta)d\beta.$$

Thus, the appearance is Poisson as desired.

APPENDIX B

$$V(c_1,c_2) := \max_p W(p;\ c_1,c_2)$$

$$:= \max_p \int_0^\infty \Bigg[ \int_0^t e^{-rs}\pi(p;\ c_1,c_2)ds$$

$$+ e^{-rt}\int_0^p \frac{\max\ \pi(z;\ c_1,c)\ \frac{f(c)}{F(p)}}{r}\,dc \Bigg] \lambda_o F(p)\exp[-\lambda_o \int_0^t F(p)ds]dt$$

Integrating and simplifying yields

$$W(p,c_1,c_2) = \frac{\pi(p;\ c_1,c_2) + \int_0^p \max_z \frac{\pi(z;\ c_1,c)}{r} f(c)\ dc}{r + \lambda_o F(p)}$$

Neglecting the case where $c_2$ is of any consequence, p will be chosen such that

$$0 = \frac{dW}{dp}(p;\ c_1,c_2)$$

$$= \frac{1}{r+\lambda_o F(p)} \Bigg[ \pi'(p;\ c_1,c_2) + \max_z \frac{\pi(z;\ c_1,p)f(p)}{r}$$

$$- \frac{[\pi(p;\ c_1,c) + \int_0^p \max_z \frac{\pi(z;\ c_1,c)f(c)}{r}]f(p)}{r+\lambda_o F(p)} \Bigg]$$

$$= \frac{1}{r+\lambda_o F(p)} \Bigg[ \pi'(p;\ c_1,c_2) + [\max_z \frac{\pi(z;\ c_1,p)}{r} - V(c_1,c)]\ f(p) \Bigg]$$

Since $\max_z \frac{\pi(z;\ c_1,p)}{r} > V(c_1,p) \geq V(c_1,c)$ for $c \leq p$ it follows that at an optimum:

$$\pi'(p;\ c_1,c_2) \leq 0$$

with equality only when $f(p) = 0$. The reason for this counter-intuitive result is easy to understand: when entry can occur only once, raising p increases the chance of attracting a high cost entrant, thereby forestalling entry by a low cost entrant.